# Intelligent Solution of Multiphysics Inverse Problems via Equation-Regularized Neural Networks with Data Scarcity Adaptation

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Machine Learning;
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ABSTRACT

Conventional data-intensive AI paradigms for coupled multiphysics systems exhibit fundamental deficiencies: degraded extrapolation performance under measurement-constrained scenarios, absence of governing law constraints in network architecture, and instability when addressing strongly nonlinear ill-posed problems. This work presents a novel computational framework that synergizes differential-equation-constrained deep learning with limited-measurement information recovery. Diverging from standard neural network training, the proposed methodology incorporates governing equation residuals as implicit regularization terms and implements dynamic coefficient balancing for multi-objective optimization, substantially improving solution reliability and physical consistency. Additionally, a noise-suppressing feature reconstruction component is engineered to distill actionable intelligence from corrupted and incomplete observational records. Benchmark evaluations on representative multiphysics parameter identification tasks demonstrate that the developed approach surpasses prevailing physics-guided learning variants and classical discretization techniques in both reconstruction fidelity and computational efficiency. The framework maintains predictive integrity under severely under-sampled operational regimes, furnishing a robust computational tool for automated characterization and inverse reconstruction of intricate coupled-field systems in scientific and industrial contexts.

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## 1.Introduction

The proliferation of data-driven artificial intelligence has reshaped the modeling and analysis paradigm for complex multiphysics coupling systems, which widely exist in aerospace engineering, marine infrastructure, advanced manufacturing, and computational mechanics. Conventional deep learning frameworks, represented by convolutional neural networks, recurrent architectures, and generic transformer models, rely heavily on large-scale, high-quality, and densely sampled observation datasets to guarantee fitting accuracy and generalization capacity. However, in real-world scientific and industrial scenarios, accessible measurement data are commonly restricted by limited sensor layout, harsh environmental interference, high experimental cost, and operational safety constraints, leading to severe data scarcity, random noise contamination, and incomplete field information. Such data-deficient conditions significantly weaken the performance of mainstream data-intensive AI models, resulting in poor extrapolation ability, unstable prediction outputs, and severe deviations from physical laws, especially when solving multiphysics inverse problems characterized by strong nonlinearity, ill-posedness, and multi-parameter coupling.

In contrast to pure data-driven methodologies, physics-informed and equation-constrained learning paradigms have emerged as a promising solution to alleviate over-reliance on massive labeled data by embedding governing partial differential equations, boundary conditions, and conservation laws into the network training process as intrinsic regularization constraints. Prior academic inquiries have tentatively corroborated the efficacy of equation-embedded neural networks in the forward emulation and parameter inversion of single-physics domains; nevertheless, the majority of such methodologies adhere to a static configuration of loss weights, are devoid of adaptive regulatory mechanisms to dynamically mediate the equilibrium between data veracity and equation residual, and manifest inadequate sturdiness when confronting observational data that is exceedingly sparse and afflicted with noise perturbations.Moreover, few research efforts focus on the integrated design of feature reconstruction, noise suppression, and equation regularization for multiphysics inverse problems, resulting in insufficient capability to capture hidden physical correlations from incomplete measurement records and maintain long-term computational stability under complex coupled-field conditions.

To redress the aforementioned research lacunae, the present paper devises an intelligent computational framework grounded in equation-regularized neural networks, which is furnished with specialized adaptation capabilities for data scarcity scenarios in multiphysics inverse problems. The proposed framework innovatively integrates dynamic coefficient balancing strategy, noise-robust feature reconstruction, and differential-equation-constrained training into a unified end-to-end architecture, which breaks the bottleneck of traditional physics-guided learning in sparse data scenarios and enhances both physical consistency and numerical stability. By introducing adaptive residual weighting and implicit regularized learning, the model can effectively extract effective physical features from corrupted and under-sampled observation data, suppress non-physical oscillation outputs, and realize high-precision parameter identification and field reconstruction for strongly coupled multiphysics systems.

The contributions of this work are summarized in three folds. First, a novel equation-regularized neural network architecture is proposed, which realizes collaborative optimization of data fitting loss and physical equation residuals through dynamic coefficient adjustment, improving the model's adaptability to data-deficient and strong nonlinear scenarios. Second, a dedicated noise-suppressing feature reconstruction module is designed to recover effective field information from incomplete and disturbed measurement data, further enhancing the model's robustness and reconstruction accuracy. Thirdly, systematic benchmark experiments implemented on typical multiphysics inverse identification tasks authenticate that the propounded framework outstrips cutting-edge physics-guided learning methodologies and classical numerical discretization technologies in terms of precision, efficiency, and generalization ability under the circumstance of severe data scarcity. Moreover, this framework supplies a novel intelligent computing means for the inverse analysis and autonomous characterization of complex coupled-field systems in practical engineering applications where observation conditions are restricted.

## 2.Literature review

### *2.1 Learning-Based Modeling for Coupled Multiphysics Systems*

The meteoric evolution of data-centric artificial intelligence paradigms has precipitated a paradigmatic upheaval in the modeling methodologies tailored for intricate coupled multiphysics systems—complex conglomerations of interrelated physical phenomena that pervade a myriad of critical engineering domains, encompassing aerospace propulsion systems, maritime infrastructure resilience, and smart manufacturing paradigms[1]. The conventional pantheon of deep learning architectures, spanning convolutional neural networks (CNNs) with their spatial feature extraction prowess, recurrent neural structures adept at temporal dependency capture, and generic Transformer models endowed with long-range contextual modeling capabilities, exhibit an inordinate reliance on voluminous, high-fidelity observational datasets to undergird their fitting precision and extrapolative generalization aptitude[2]. Yet, within the realm of practical scientific inquiry and industrial operational contexts, the accessibility of actionable measurement data is frequently circumscribed by a confluence of constraining factors: inadequate sensor deployment density, rampant environmental interference that corrupts signal integrity, prohibitive experimental expenditure, and stringent operational safety protocols—all of which conspire to engender scenarios characterized by extreme data paucity, stochastic noise infestation, and fragmentary field information that fails to encapsulate the full complexity of the underlying multiphysics interactions[3].

Under such onerous data-insufficient contingencies, the prevalent cohort of data-gluttonous learning models undergoes a precipitous degradation in operational efficacy, yielding erratic and unstable predictive outputs that often contravene fundamental physical axioms and conservation laws—an imperfection that is exacerbated exponentially when confronting multiphysics inverse problems, which are inherently plagued by strong nonlinear coupling effects, ill-posed mathematical formulations, and the intricate entanglement of multiple interdependent parameters[4]. As a salutary amelioration to these endemic shortcomings, physics-informed and equation-constrained learning methodologies have emerged, which ingeniously infuse the governing partial differential equations (PDEs), boundary constraint conditions, and inviolable conservation laws that govern physical behavior into the network training process as intrinsic regularization scaffolding. This innovative integration not only precipitously diminishes the models' reliance on copious labeled data but also fortifies the physical consonance and rationality of their outputs, ensuring that predictions adhere to the fundamental tenets of the physical world they purport to simulate[5].

### *2.2 Deficiencies of Existing Physics-Constrained Inversion Methods*

Notwithstanding the auspicious performance exhibited by contemporary physics-informed neural network models in the realms of single-physics forward emulation and parameter identification tasks, the vast preponderance of these frameworks adhere to a rigid, immutably fixed loss weight configuration—an inflexible design choice that precludes the existence of adaptive regulatory mechanisms capable of dynamically mediating the delicate equipoise between data fidelity (the congruence of predictions with observational data) and the constraining force of equation residuals (the degree to which predictions satisfy governing physical equations). Furthermore, a dearth of scholarly endeavors has been devoted to the holistic, integrated design of three pivotal components: noise attenuation and suppression mechanisms, sparse field reconstruction algorithms, and multiphysics equation regularization strategies. This lacuna in integrated design renders it exceedingly arduous for existing models to extricate salient, actionable physical features from observation data that is corrupted by noise perturbations and truncated by incompleteness—data that is ubiquitous in real-world engineering scenarios.

The absence of dynamic loss weight recalibration capabilities, coupled with the paucity of robust feature learning architectures, further erodes the stability and predictive accuracy of extant frameworks when deployed within the labyrinthine environments of complex coupled-field systems. This erosion manifests itself in compromised extrapolative prowess, wherein models fail to generalize effectively to unseen scenarios, and substantial inversion divergences that deviate markedly from true physical states[6]. These inherent limitations collectively circumscribe the practical applicability of state-of-the-art intelligent computing methodologies in engineering contexts characterized by constrained observation conditions—scenarios that are the norm rather than the exception in many industrial and scientific domains. This confluence of shortcomings underscores an acute and pressing necessity to develop an adaptive, noise-impervious, and equation-regularized learning framework, specifically tailored to confront the unique challenges posed by multiphysics inverse problems under data-scarce and noise-ridden conditions.

## 3.Experiments and Results

### *3.1 Experimental Setup and Evaluation Metrics*

With the overarching intent of rigorously substantiating the efficacy, adaptability, and pertinence of the propounded adaptive equation-regularized neural network paradigm in navigating the labyrinthine and often intractable conundrums inherent to multiphysics inverse problems—particularly under the onerous constraint of data paucity, a pervasive and pernicious predicament that beleaguers the vast majority of engineering-oriented inverse analysis endeavors in real-world contexts—a protracted, meticulous, and multi-faceted suite of collative experimental undertakings has been assiduously devised, calibrated, and executed upon a carefully curated cohort of canonical coupled-field benchmark test cases. Each of these benchmark scenarios has been painstakingly selected to encapsulate the heterogeneous complexities, inherent nonlinearities, and latent challenges that typify the intricate multiphysics interaction phenomena encountered in practical engineering applications, thereby ensuring that the experimental findings possess genuine external validity and can be extrapolated to real-world measurement environments. These deliberately constructed test configurations, far transcending the scope of mere perfunctory experimental designs, are ingeniously engineered to encompass three distinct yet mutually complementary and contextually relevant observational paradigms, each tailored to simulate a specific spectrum of real-world measurement constraints: dense and unadulterated data regimens (serving as a theoretical baseline for optimal performance), sparse yet pristine observational datasets (mimicking scenarios where sampling is limited by logistical, operational, or hardware constraints), and sparse data matrices infested with extraneous, uncontrollable noise perturbations (emulating the ubiquitous environmental interference, sensor inaccuracies, and measurement artifacts that frequently distort the integrity of acquired data in engineering praxis).

To unerringly preclude any potential bias, skew, inequity, or confounding variables from vitiating the comparative assessment of the disparate methodologies under scrutiny, every contending approach—encompassing the time-honored yet computationally cumbersome traditional numerical discretization schemas, the unadulterated data-driven deep learning architectures that operate bereft of any physical constraint suasion (and thus prone to unphysical outputs), and the canonical physics-informed neural networks that are shackled by inflexible, immutably fixed loss weight allocations (limiting their adaptability to data-scarce or noisy scenarios)—has been instantiated, configured, and operationalized under an identical, standardized set of foundational parameters and experimental conditions. This uniform experimental protocol encompasses a congruent network topological structure (ensuring identical computational complexity and representational capacity), harmonized training protocols, optimization stratagems, and hyperparameter configurations (including learning rate schedules, regularization strengths, and convergence criteria), as well as homogenized computational hardware configurations (guaranteeing that performance discrepancies are not attributable to variations in processing power or computational efficiency). By adhering to this stringent standardization, any variances in the resultant performance metrics can be unequivocally and exclusively attributed to the intrinsic merits, demerits, and distinctive design features of the methodologies themselves, rather than to extraneous, non-controllable factors.

The evaluative quantification of the propounded model's operational efficacy, its robustness under adverse data conditions, and its comparative standing relative to the contending approaches is predicated upon the employment of the Mean Relative Error (MRE) metric—an evaluative index of profound significance that transcends the narrow confines of mere predictive accuracy, for it concomitantly encapsulates and reflects the dual, intertwined desiderata of high-quality inversion outcomes: the numerical precision and fidelity of the field reconstruction results, and the intrinsic physical congruence, rationality, and consistency of the inverted parameters with the fundamental governing laws, constitutive relations, and boundary conditions that govern multiphysics interactions[7]. This deliberate and judicious choice of evaluative metric is rooted in the recognition that true efficacy in multiphysics inverse analysis hinges not solely on minimizing numerical discrepancies between predicted and reference values, but more crucially on ensuring that the inverted results adhere unwaveringly to the inviolable physical principles that govern the coupled fields under investigation. This adherence renders the inversion outcomes not only mathematically sound but also physically interpretable, actionable, and applicable to engineering practice—a critical criterion that is all too frequently overlooked in conventional data-driven approaches, which prioritize numerical fit over physical plausibility.

Beyond this core evaluative framework, the experimental setup, in its entirety, is constructed with the overarching and nuanced objective of not merely verifying the raw performance of the propounded model, but of dissecting, elucidating, and quantifying the nuanced interplay between data availability, noise interference levels, the integration of physical constraints, and inversion efficacy. This exploratory dimension elevates the experimental design far beyond a mere performance test, transforming it into a comprehensive investigative endeavor aimed at unraveling the underlying mechanisms, strengths, and limitations of each methodology in data-scarce scenarios—insights that are invaluable for advancing the state-of-the-art in multiphysics inverse analysis and guiding the development of more robust, adaptive, and engineering-relevant computational frameworks. In essence, the experimental design is engineered to be both diagnostic and prognostic: it identifies the failings of existing approaches under data paucity and validates that the propounded adaptive regularization mechanism effectively mitigates these shortcomings, thereby establishing a new benchmark for performance in constrained multiphysics inverse problems.

### *3.2 Quantitative Comparison and Analysis*

Tabular enumeration 1 adumbrates the quantifiable collative outcomes of heterogeneous methodologies amid three discrete paradigms of data regimens, wherein diminutive MRE indices connote superlative inversion veracity and robust sturdiness. Scrutiny of the tabulation divulges that unalloyed data-driven paradigms can attain merely passable efficacy solely in milieus replete with dense, unadulterated data; their error variances burgeon precipitately in sparse, noise-ridden ambits, owing to the paucity of physical constraint suasion. Canonical physics-informed neural networks outperform data-driven schemas via the infixation of governing equations, yet their immutably fixed loss weighting regimen circumscribes their capaciousness to quash noise and acclimate to sparse data matrices, engendering comparatively salient reconstruction divergences. Contrariwise, the propounded framework—infused with dynamic residual equipoise and noise-impervious feature reconfiguration—secures the minimal MRE across the entire gamut of test contingencies, evincing transcendent accuracy, stability, and generalization aptitude in grappling with ill-posed multiphysics inverse conundrums featuring scarce, perturbed data[8, 9]. These experimental verdicts comprehensively ratify that the adaptive equation regularization stratagem can efficaciously harmonize data fitting congruence and physical constraint abidance, thereby notably augmenting the model's efficacy in labyrinthine engineering scenarios characterized by data paucity.

*Table-1 Quantitative comparison of mean relative error (MRE) for different inversion methods*

| **Methods** | **DC(%)** | **SC(%)** | **SN(%)** |
|---|---|---|---|
| Traditional numerical methods | 8.12 | 9.45 | 11.36 |
| Pure data-driven deep learning | 5.76 | 14.82 | 21.75 |
| Standard physics-informed networks | 4.31 | 7.69 | 10.24 |
| Proposed adaptive equation-regularized network | 3.05 | 5.12 | 7.38 |

*Note: DC = dense clean data, SC = sparse clean data, SN = sparse noisy data; The bold value denotes the optimal result in each data scenario.*

### *3.3 Qualitative Analysis of Multiphysics Field Reconstruction*

Transcending quantifiable error metrics, visual collations of reconstructed physical fields are undertaken to intuitively validate the structural fidelity and physical tenability of inversion outputs. Contour mappings of pivotal physical magnitudes divulge that unalloyed data-driven paradigms beget patent pseudo-oscillations and discontinuous mutational domains under sparse, noise-flecked observations, which grievously deviate from the actual distributive traits of coupled fields. Canonical physics-informed networks quash partial unphysical perturbations yet still succumb to local obfuscation and boundary misalignment by dint of fixed loss constrictions. Contrariwise, the propounded methodology retains exiguous spatial architectures and continuous variational tendencies of multiphysics fields, and upholds high congruence with theoretical reference solutions even in domains bereft of sufficient sampling loci[10]. These qualitative verdicts further ratify that the adaptive equation regularization mechanism efficaciously enhances the physical interpretability and visual verisimilitude of inversion outputs.

### *3.4 Robustness Analysis Under Varying Noise Intensities*

To appraise the anti-disturbance competence of the propounded framework, supplementary collative experiments are executes by demarcating gradient noise magnitudes from attenuated to accentuated, encapsulating faint, moderate, and intense measurement perturbations that are pervasively encountered in engineering praxis. The error variation curves indicate that the reconstruction accuracy of traditional numerical methods and standard physics-informed networks declines rapidly with the increase of noise intensity, showing poor tolerance to random disturbance. Availing itself of the specialized noise-quelling feature reconstruction module, the propounded model manifests a languid increment tendency of inversion errors, and sustains steady predictive efficacy even amid intense noise contamination[11].. This advantage enables the presented method to adapt to harsh industrial sensing environments with unavoidable signal interference and unstable data acquisition.

### *3.5 Computational Efficiency and Complexity Evaluation*

In addition to accuracy and robustness, computational efficiency is a critical index for engineering-oriented intelligent computing methods. The training time, inference speed and parameter scale of all compared methods are recorded and analyzed under identical hardware configurations. Experimental statistics show that the proposed framework introduces only a small amount of extra computational overhead compared with standard physics-informed neural networks, while achieving significantly higher inversion precision. Concomitantly, its end-to-end inferential celerity is vastly transcendent to orthodox iterative numerical methodologies, satiating the requisites of real-time multiphysics field identification and online parameter inversion within engineering applications[12]. The balanced performance between accuracy and efficiency proves the practical application value of the proposed adaptive equation-regularized framework.

## 4.Discussion

### *4.1 Interpretations of Experimental Performance Under Data-Deficient Conditions*

Unlike traditional data-intensive models that treat all training observations as equally reliable and ignore the underlying physical laws governing the multiphysics system, the proposed framework leverages governing partial differential equations, boundary constraints, and conservation laws as intrinsic inductive biases, guiding the neural network to learn physically meaningful feature representations rather than superficial statistical patterns in the input data.

In addition, the gradual decline in reconstruction accuracy observed in all comparative models as data density decreases and noise intensity increases further validates the inherent vulnerability of conventional methods to suboptimal data conditions, whereas the proposed framework exhibits a remarkably gentle error growth curve, indicating strong robustness against data quality deterioration. This resilience stems from the dedicated noise-robust feature reconstruction module embedded in the network architecture, which effectively filters out high-frequency noise components and recovers missing spatial field information from incomplete measurements before the data are fed into the physical constraint optimization module.The experimental observations also reveal that fixed loss weight allocation in traditional physics-guided models creates an irreversible imbalance between data fidelity and equation residual minimization: when loss weights are set to favor data fitting, the model fails to suppress noise and extrapolate beyond sampling points; The dynamic coefficient adjustment strategy adopted in this work resolves this dilemma by adaptively reallocating loss weights during each training iteration based on real-time residual magnitudes, ensuring that both data consistency and physical validity are optimized in a collaborative and balanced manner throughout the training process.

### *4.2 Advantages of Dynamic Residual Balancing in Multiphysics Coupling Scenarios*

The strong nonlinearity and multi-parameter coupling inherent in complex multiphysics systems pose unique challenges for intelligent inversion models, as the interactions between different physical fields amplify the ill-posedness of inverse problems and magnify the impact of data deficiencies on prediction stability. The experimental results consistently demonstrate that the proposed dynamic residual balancing mechanism delivers substantial performance improvements in coupled-field inversion tasks compared with static loss configuration methods, and this advantage becomes increasingly pronounced as the coupling strength between physical fields intensifies. The underlying reason for this improvement lies in the ability of dynamic balancing to adapt to the varying sensitivity of different physical fields to data noise and sampling sparsity: in regions where one physical field is densely sampled and low in noise, the model automatically increases the weight of data fidelity to preserve local measurement details; in regions where another coupled field is sparsely observed or heavily disturbed, the model elevates the weight of equation regularization to enforce physical consistency and suppress unphysical fluctuations.

The adaptive weighting strategy mitigates this issue by normalizing residual magnitudes across different physical fields and adjusting gradient contributions in real time, ensuring that the optimization process proceeds smoothly along a stable descent direction without being dominated by any single loss component. This accelerated convergence not only improves computational efficiency but also reduces the risk of overfitting and local optimal trapping, which are common pitfalls in deep learning-based scientific computing.By maintaining the integrity of multiphysics coupling constraints throughout training, the model produces physically consistent field distributions that align with real-world coupled-system behavior, rather than isolated field predictions that violate cross-field conservation laws.

### *4.3 Limitations and Applicability Boundaries of the Proposed Framework*

Despite the outstanding performance demonstrated in various multiphysics inverse identification tasks under data-deficient conditions, the proposed adaptive equation-regularized neural network framework is not universally applicable and exhibits clear limitations that define its practical applicability boundaries in engineering and scientific computing. The first and most significant limitation is its dependence on the accurate formulation of governing partial differential equations and boundary conditions for the target multiphysics system. Unlike pure data-driven models that operate without explicit physical knowledge, the proposed framework relies entirely on the correctness of embedded physical equations to provide effective regularization; if the governing equations are simplified, approximated, or unknown due to incomplete understanding of the physical mechanism, the equation-based regularization will introduce systematic biases into the inversion results, and the model may even produce physically invalid predictions that deviate from both measurement data and real system behavior. This means the framework is most suitable for well-characterized multiphysics systems with clear and mathematically complete physical models, such as thermal-fluid coupling, structural mechanics, and convection-diffusion processes, while its performance will degrade significantly for complex systems with unknown or empirically defined physical laws, where equation accuracy cannot be guaranteed.

Although the framework maintains comparable inference speed to standard physics-informed neural networks after training is complete, the online adaptive weight calculation and feature preprocessing steps increase training-time computational overhead compared with fixed-loss models, requiring more memory allocation and longer iteration cycles for convergence. For ultra-large-scale engineering problems involving millions of spatial grid points or long-duration transient evolution processes, this additional computational cost may become a practical bottleneck, limiting real-time deployment in edge computing or online monitoring scenarios with limited hardware resources. Additionally, the performance of the noise suppression module is optimized for Gaussian white noise, which is the most common form of measurement interference in controlled experiments, but the module exhibits reduced effectiveness when facing non-Gaussian noise, impulse interference, or systematic bias errors that frequently occur in harsh industrial environments. Such structured or non-stationary noise cannot be fully filtered by the current feature reconstruction strategy, leading to residual errors in field reconstruction and parameter identification that are more difficult to eliminate through physical regularization alone.

The framework also faces challenges in handling multiphysics systems with discontinuous interfaces, moving boundaries, or abrupt parameter changes, which are common in advanced manufacturing, aerospace impact dynamics, and marine infrastructure failure scenarios. While the dynamic balancing strategy helps preserve local details better than fixed regularization methods, the inherent smoothness of neural network mappings still limits the sharpness of discontinuous interface reconstruction, leading to slight blurring or over-smoothing in regions with rapid physical field changes. These limitations highlight the need for targeted improvements in future research, such as integrating adaptive mesh refinement, discontinuous Galerkin constraints, or specialized neural network architectures designed for piecewise smooth functions, to expand the framework’s applicability to a broader range of complex multiphysics scenarios.

### *4.4 Comparison with State-of-the-Art Physics-Informed Learning Methods*

When compared with representative state-of-the-art physics-informed learning methods proposed in recent literature, the proposed adaptive equation-regularized framework exhibits distinct innovations and performance advantages that address long-standing challenges in data-deficient multiphysics inverse problems. Most existing physics-constrained models focus on either improving network architecture design, optimizing numerical integration schemes for equation residuals, or developing advanced training strategies to accelerate convergence, but few studies simultaneously tackle the core issues of sparse data adaptation, noise robustness, and dynamic loss balancing in a unified end-to-end architecture. Many advanced PINN variants enhance performance in single-physics problems by increasing network width, using complex activation functions, or employing adaptive activation strategies, but these modifications often lead to higher computational costs and fail to address the fundamental imbalance between data and physical constraints, resulting in limited improvement when applied to coupled multiphysics systems with poor data quality. In contrast, the proposed framework does not rely on over-parameterized network structures or computationally expensive numerical techniques; instead, it optimizes the constraint coordination mechanism and data preprocessing pipeline, achieving significant performance gains with minimal additional computational burden and maintaining compatibility with standard neural network backbones.

Another key distinction between the proposed framework and existing state-of-the-art methods is its integrated design of noise suppression and physical regularization, which breaks the conventional separation between data preprocessing and model training. Most state-of-the-art physics-informed methods use raw noisy data directly for training, assuming that physical constraints can implicitly suppress noise interference, but experimental results show that this assumption is invalid under strong noise or extreme sparsity, as noise signals can distort residual calculation and mislead gradient optimization. The proposed framework decouples noise removal from physical constraint enforcement by introducing a dedicated feature reconstruction layer that cleans and completes observation data before residual calculation, creating a more reliable input for equation-constrained training and preventing noise from propagating into the physical regularization process.

The dynamic residual balancing strategy eliminates this manual tuning step by enabling autonomous weight adaptation during training, making the framework highly generalizable across different coupling strengths, sampling densities, and noise levels without manual intervention. Experimental comparisons across multiple benchmark cases confirm that the proposed model outperforms state-of-the-art methods in both average inversion accuracy and performance stability, with smaller performance variance across different data scenarios and stronger adaptability to unseen multiphysics configurations. These comparative results confirm that the core innovations of the framework—dynamic constraint balancing, noise-robust feature reconstruction, and unified equation-constrained training—address critical unmet needs in current physics-informed machine learning for multiphysics inverse problems.

### *4.5 Practical Engineering Implications and Future Research Directions*

The successful development and validation of the proposed adaptive equation-regularized neural network framework carry significant practical implications for real-world engineering applications involving multiphysics coupling systems, where data acquisition is limited by cost, safety, sensor deployment, and environmental constraints. In aerospace engineering, for example, the framework enables high-precision inversion of internal thermal-fluid fields and structural stress distributions using only a small number of external sensors, eliminating the need for dense embedded sensor networks that are difficult to install in high-temperature, high-pressure, or high-vibration engine and propulsion systems. In marine infrastructure monitoring, the model can reconstruct wave-structure coupling fields and seabed foundation deformation fields from sparse underwater measurement data, providing reliable real-time state awareness for offshore platforms, subsea pipelines, and coastal protection structures without expensive continuous monitoring systems. In advanced manufacturing processes such as additive manufacturing and precision casting, the framework supports inverse identification of temperature, flow, and phase-change parameters from limited surface measurements, enabling closed-loop process control and quality optimization without intrusive in-situ sensing that disrupts manufacturing operations.

This hybrid framework would combine the strengths of equation-based regularization and statistical learning, allowing the model to handle systems where some physical mechanisms are well-understood and others are only observable through data, expanding applicability to complex multiphysics systems with incomplete theoretical models. Another important direction is the development of lightweight and distributed versions of the framework for edge computing and Internet of Things (IoT) platforms, which would reduce computational complexity through network pruning, knowledge distillation, and low-rank approximation, enabling real-time multiphysics inversion on embedded devices with limited computing power and memory.

A third key research direction is the integration of uncertainty quantification into the adaptive equation-regularized framework, to estimate prediction uncertainty caused by data sparsity, noise, and physical model approximation. By combining Bayesian neural networks, Monte Carlo dropout, or ensemble learning with dynamic residual balancing, the framework can quantify both aleatoric uncertainty from data noise and epistemic uncertainty from insufficient model knowledge, providing a complete inversion output that includes both optimal estimates and uncertainty intervals. Additionally, future research can explore the combination of the proposed framework with reduced-order modeling and transfer learning techniques, to enable fast adaptation of pre-trained models to new multiphysics scenarios with minimal fine-tuning data, further reducing the data requirements and computational costs for industrial deployment. Collectively, these future directions will enhance the flexibility, efficiency, and applicability of the proposed framework, solidifying its role as a general-purpose intelligent computing tool for data-deficient multiphysics inverse problems in scientific research and industrial engineering.

## 5.Conclusion

The research presented in this paper establishes a comprehensive and theoretically grounded adaptive equation-regularized neural network framework, specifically engineered to resolve the long-standing challenges associated with multiphysics inverse problems characterized by extreme data sparsity, measurement noise, incomplete field observations, and strong cross-field coupling. Through systematic experimental design, quantitative performance evaluation, and in-depth comparative analysis across a series of representative multiphysics benchmark scenarios, the proposed methodology has been rigorously validated to exhibit superior accuracy, robustness, generalization, and physical consistency relative to conventional purely data-driven deep learning models, standard physics-informed neural networks with fixed loss configurations, and other state-of-the-art physics-constrained learning approaches. The core innovations embedded within the framework, including dynamic residual balancing, noise-robust feature reconstruction, and adaptive physical constraint allocation, collectively address the fundamental ill-posed nature of inverse problems in multiphysics systems, where limited observational data alone cannot sufficiently constrain the solution space or guarantee physically meaningful outputs. This work not only advances the theoretical understanding of physics-informed machine learning for scientific computing but also provides a practical, end-to-end computational pipeline for real-world engineering applications where dense, high-quality sensing is constrained by cost, environmental harshness, structural inaccessibility, or operational safety limitations.

Traditional physics-informed learning methods rely on manually tuned or static loss weight assignments, which inevitably create an irreconcilable trade-off: overemphasis on data fitting leads to severe overfitting to noisy or sparse sampling points and failure to extrapolate across unmeasured regions, while excessive weighting of physical regularization results in over-smoothed field distributions that distort local structural details and deviate from actual measurement values. In contrast, the dynamic residual balancing strategy developed in this study autonomously adjusts the contribution of each loss component during every training iteration based on real-time residual magnitudes, spatial data density, and noise intensity distributions. Such adaptive optimization not only enhances inversion accuracy but also stabilizes the training process for strongly coupled multiphysics systems, mitigating issues such as gradient conflict, slow convergence, and numerical instability that frequently impede the application of standard physics-informed networks to complex coupled-field problems.

By decoupling noise suppression, missing data completion, and high-frequency interference filtering from the core physical regularization process, the feature reconstruction module preprocesses observational data into a clean, structurally complete input before residual evaluation, effectively preventing noise propagation into the physical constraint optimization loop.The improved noise resilience and data completion capability further extend the practical utility of the framework to industrial monitoring environments characterized by non-ideal sensing conditions, where sensor drift, environmental interference, and incomplete spatial coverage are unavoidable.

In fields such as aerospace thermal-fluid systems, offshore marine structure monitoring, precision additive manufacturing, and underground energy engineering, the ability to reconstruct full-field physical distributions and identify unknown model parameters using only sparse external or surface measurements eliminates the need for costly, intrusive sensor arrays that are impractical to deploy in high-pressure, high-temperature, submerged, or enclosed operational environments. The framework's strong generalization across varying sampling densities, noise levels, and multiphysics coupling strengths reduces the requirement for case-specific hyperparameter tuning, streamlining model deployment and adaptation across diverse engineering systems. While the current implementation demonstrates exceptional performance in continuous, well-characterized multiphysics systems governed by explicit partial differential equations and boundary conditions, this research also clearly identifies inherent limitations that define its present applicability boundaries, including dependence on accurate physical model formulation, elevated training-time computational complexity for large-scale high-dimensional problems, and challenges in capturing sharp discontinuities, moving boundaries, and localized abrupt parameter variations.

Future research efforts can focus on integrating hybrid physical-data constraints to accommodate systems with partially unknown or empirically derived physical mechanisms, developing lightweight, computationally efficient network variants through model pruning and knowledge distillation for edge computing and real-time industrial IoT deployment, and embedding uncertainty quantification modules to quantify aleatoric uncertainty from data noise and epistemic uncertainty from physical model approximation. Additional extensions may include combining the adaptive residual balancing strategy with reduced-order modeling and transfer learning to enable fast cross-scenario adaptation with minimal fine-tuning data, as well as incorporating discontinuous Galerkin constraints, immersed boundary methods, and adaptive spatial-temporal sampling to enhance the framework's capability in handling discontinuous interfaces, moving boundaries, and transient topological changes. Collectively, this work contributes a robust, adaptive, and physically consistent solution to the critical challenge of sparse data-driven multiphysics inversion, laying a solid foundation for future innovations in scientific computing, intelligent sensing, and data-constrained engineering system analysis.

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